

# Wormhole and C-field: Revisited

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## Abstract

Recently, Rahaman et al [ Nuovo.Cim 119B, 1115(2004)] have shown that the static spherically symmetric solutions in presence of C-field give rise to wormhole geometry. We highlight some of the characteristics of this wormhole, which have not been considered in the previous study.

In recent times, the wormhole has become very popular as because it could allow for interstellar distances to be traveled in very short times. In a seminal paper, Morris and Thorne [MT][1] have shown that wormholes are solutions of the Einstein's equations that have two regions connected by a throat. They observed that it is the solutions of Einstein's equations that shared the violation of null energy condition. This bizarre form of matter that characterized the above stress energy tensor is known as exotic matter. If, some how, an advanced engineers able to manufacture the exotic matter, then it would be possible to construct a wormhole. If wormhole could be constructed, the faster than light travel would be possible. In other words, time machine must be constructed. There are different ways of evading these unexpected matter energy source. Most of these attempts focus on alternative theories of gravity or phantom energy ( i.e. cosmic fluid which is responsible for the accelerated expansion of the Universe ). Several authors have explained wormholes in scalar tensor theory of gravity in which scalar field may play the role of exotic matter [2-7]. Last few years, physicists have discussed the physical properties and characteristics of traversable wormholes by taking phantom energy as source[8-12]. Long ago, since 1966, Hoyle and Narlikar [HN] proposed an alternative theory of gravity known as C-field theory [13]. HN adopted a field theoretic approach introducing a massless and chargeless scalar field C in the Einstein-Hilbert action to account for the matter creation.

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The complete action functional describing C-field, matter and gravity is taken as

$$A = \frac{1}{16\pi G} \int R \sqrt{-g} d^4x - \frac{1}{2} f \int C_i C^i \sqrt{-g} d^4x + A_1 + A_{matter} \quad (1)$$

Here  $C_i = (\frac{\partial C}{\partial x_i})$  and  $f > 0$  is a coupling constant. The action  $A_1$  can be taken to be  $\sum_n \int C_i d^4x_n^i$  where the coordinates  $x_n^i$  represent the world line of the  $n^{th}$  particle. On varying the above action w.r.t.  $g^{ab}$ , one can get the usual Einstein equation with the addition of C-field energy density

$$R^{ab} - \frac{1}{2} g^{ab} R = -8\pi G [T^{ab} - f C^a C^b + \frac{1}{2} f g^{ab} C^i C_i] \quad (2)$$

Here,  $T_{ab}$  is the matter tensor.

A C-field generated by a certain source equation, leads to interesting change in the cosmological solution of Einstein field equations. Several authors, have studied cosmological models [14] and topological defects [15] in presence of C-field. Since, C-field theory is a scalar tensor theory, so it is interesting to search whether the creation field C may play the role of exotic matter that is required to get wormhole solution. Recently, Rahaman et al [16] have pointed out that a spherically symmetric vacuum solutions to the C-field theory give rise to a wormhole. In this report, we would like to mention some of the characteristics ( i.e. matching with Schwarzschild metric, traversability etc ) of the C-field wormhole, which have not been considered in the previous study.

Let us consider the static spherically symmetric metric as

$$ds^2 = -e^\nu dt^2 + e^\mu dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3)$$

The independent field equations for the metric (3) are

$$e^{-\mu} \left[ \frac{1}{r^2} - \frac{\mu'}{r} \right] - \frac{1}{r^2} = 4\pi G f e^{-\mu} (C')^2 \quad (4)$$

$$e^{-\mu} \left[ \frac{1}{r^2} + \frac{\nu'}{r} \right] - \frac{1}{r^2} = -4\pi G f e^{-\mu} (C')^2 \quad (5)$$

$$e^{-\mu} \left[ \frac{1}{2} (\nu')^2 + \nu'' - \frac{1}{2} \mu' \nu' + \frac{1}{r} (\nu' - \mu') \right] = 8\pi G f e^{-\mu} (C')^2 \quad (6)$$

The solutions are given by [16]

$$e^\nu = \text{constant} \quad (7)$$

$$e^{-\mu} = 1 - \frac{D}{r^2} \quad (8)$$

$$C = \frac{1}{\sqrt{(4\pi G f)}} \sec^{-1} \frac{r}{\sqrt{(D)}} + C_0 \quad (9)$$

where  $D$  and  $C_0$  are integration constants. We note that the creation field becomes constant and equal to  $C_0$  when  $r \rightarrow \infty$ .

Thus the metric (3) can be written in Morris-Thorne canonical form as

$$ds^2 = -e^\nu dt^2 + \frac{dr^2}{[1 - \frac{b(r)}{r}]} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (10)$$

Here,  $b(r) = \frac{D}{r}$  is called shape function and  $e^\nu = \text{redshift function} = \text{constant}$ .

The shape function is depicted in fig.1.

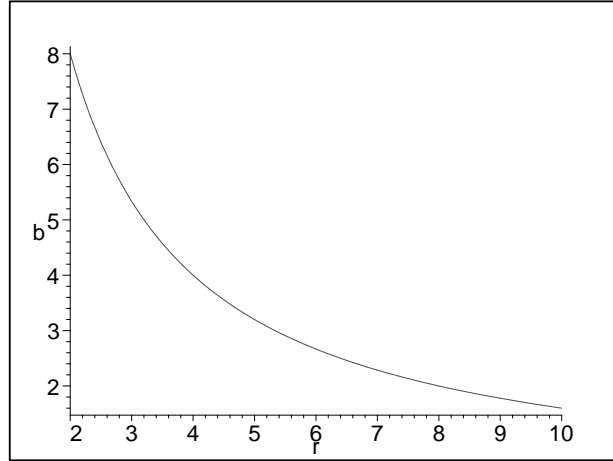


Figure 1: The shape function of the wormhole

We notice that the square root of the integration constant  $D$  indicates the position where the throat of the wormhole occurs i.e. at  $r = r_0 = \sqrt{D} > 0$ . One can note that since now  $r \geq r_0 > 0$ , there is no horizon. Now we match the interior wormhole solution to the exterior Schwarzschild solution ( in the absence of C-field ). To match the interior to the exterior, we impose the continuity of the metric coefficients,  $g_{\mu\nu}$ , across a surface,  $S$ , i.e.  $g_{\mu\nu}|_{S} = g_{\mu\nu}|_{S}$ .

[ This condition is not sufficient to different space times. However, for space times with a good deal of symmetry ( here, spherical symmetry ), one can use directly the field equations to match. Actually, if the metric coefficients are not differentiable and affine connections are not continuous at the junction then one has to use the second fundamental forms associated with the two sides of the junction surface[17] ]

The wormhole metric is continuous from the throat,  $r = r_0$  to a finite distance  $r = a$ . Now we impose the continuity of  $g_{tt}$  and  $g_{rr}$ ,

$$g_{tt(int)}|_S = g_{tt(ext)}|_S$$

$$g_{rr(int)}|_S = g_{rr(ext)}|_S$$

at  $r = a$  [ i.e. on the surface S ] since  $g_{\theta\theta}$  and  $g_{\phi\phi}$  are already continuous.

The continuity of the metric then gives generally

$$e^\nu_{int}(a) = e^\nu_{ext}(a) \text{ and } e^\mu_{int}(a) = e^\mu_{ext}(a).$$

Hence one can find

$$e^\nu = (1 - \frac{2GM}{a}) \quad (11)$$

$$\text{and } 1 - \frac{b(a)}{a} = (1 - \frac{2GM}{a}) \text{ i.e. } b(a) = 2GM$$

$$\text{This implies } \frac{D}{a} = 2GM$$

Hence,

$$a = \frac{D}{2GM} \quad (12)$$

$$\text{i.e. matching occurs at } a = \frac{D}{2GM}.$$

The interior metric  $r_0 < r \leq a$  is given by

$$ds^2 = -[1 - \frac{D}{a^2}]dt^2 + \frac{dr^2}{[1 - \frac{D}{r^2}]} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (13)$$

The exterior metric  $a \leq r < \infty$  is given by

$$ds^2 = -[1 - \frac{D}{ar}]dt^2 + \frac{dr^2}{[1 - \frac{D}{ar}]} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (14)$$

Since wormhole is not a black hole, so we have to impose the condition  $a > 2GM$ .

In recent, Das and Kar [18] have done an interesting work where they have shown that this geometry ( corresponding to eq.13 ) , can also be obtained with tachyon matter as a source term in the field equations and a positive cosmological constant.

The axially symmetric embedded surface  $z = z(r)$  shaping the Wormhole's spatial geometry is a solution of

$$\frac{dz}{dr} = \pm \frac{1}{\sqrt{\frac{r}{b(r)} - 1}}. \quad (15)$$

One can note from the definition of wormhole that at  $r = r_0$  (the wormhole throat) eq.15 is divergent i.e. embedded surface is vertical there.

The embedded surface (solution of eq.15) in this case is ,

$$z = \sqrt{D} \cosh^{-1} \frac{r}{\sqrt{D}}. \quad (16)$$

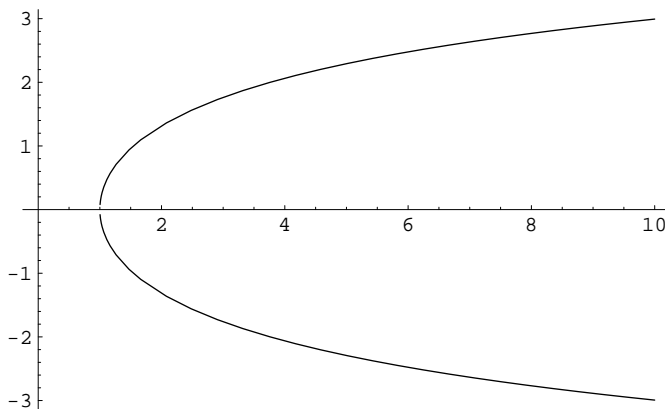


Figure 2: The embedding diagram of the wormhole

One can see embedding diagram of this wormhole in Fig.2. The surface of revolution of this curve about the vertical z axis makes the diagram complete ( see Fig.3 ).

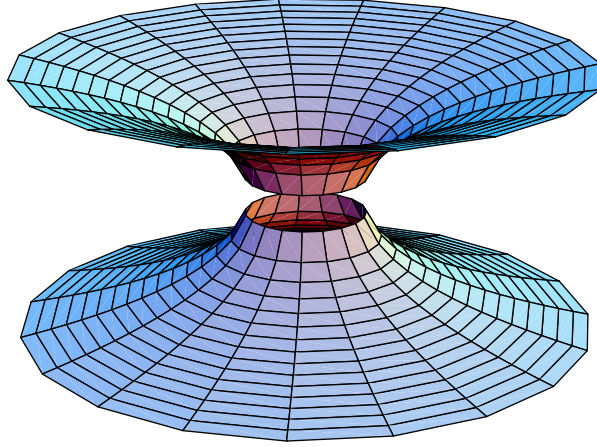


Figure 3: The full visualization of the surface generated by the rotation of the embedded curve about the vertical z axis

According to Morris and Thorne [1], the ' $r$ ' co-ordinate is ill-behaved near the throat, but proper radial distance

$$l(r) = \pm \int_{r_0^+}^r \frac{dr}{\sqrt{1 - \frac{b(r)}{r}}} \quad (17)$$

must be well behaved everywhere i.e. we must require that  $l(r)$  is finite throughout the space-time.

In this model,

$$l(r) = \pm \sqrt{r^2 - D}. \quad (18)$$

This is a well behaved coordinate system. The radial distance is positive above the throat (our Universe) and negative below the throat (other Universe). At very large distance from the throat, the embedding surface becomes flat  $\frac{dz}{dr}(l \rightarrow \pm\infty) = 0$  corresponding to the two asymptotically flat regions ( $l \rightarrow +\infty$  and  $l \rightarrow -\infty$ ), which the wormhole connects.

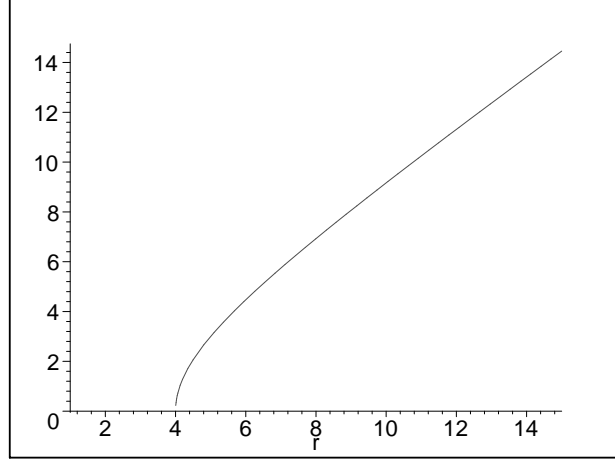


Figure 4: Diagram of the radial proper distance (  $D = 16$  )( upper half)

The radial proper distance is measured from  $r_0$  to any  $r > r_0$ . Note that on the throat  $r = r_0$ ,  $l = 0$ .

Now we will focus to the traversability condition of a human being. It is necessary that the tidal accelerations between two parts of the traveller's body, separated by say, 2 meters, must less than the gravitational acceleration at earth's surface  $g_{earth}$  (  $g_{earth} \approx 10m/s^2$  ).

According to MT [1], one obtains the following inequality for tangential tidal acceleration ( assuming  $\nu' = 0$  ),

$$| \frac{\beta^2}{2r^2} (\frac{v}{c})^2 (b' - \frac{b}{r}) | \leq \frac{g_{earth}}{2c^2 m} \approx \frac{1}{10^{16} m^2} \quad (19)$$

with  $\beta = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$  and  $v$  being the traveller's velocity.

For  $v \ll c$ , we have  $\beta \approx 1$  and substituting the expression  $b(r)$  given in (10), one gets,

$$\frac{v}{c} < \frac{r^2}{\sqrt{D} 10^8 m} \quad (20)$$

If we consider the traveller velocity  $v = .005c$  at the throat  $r = r_0$ , one finds,  $r_0 \approx 5 \times 10^5 m$ .

Taking into account equation (12) for asymptotically flat spacetimes, the region of matter distribution will extend to  $a = \frac{25 \times 10^{10}}{2GM}$ .

One can note that, by choosing  $a$ , we find the value of the wormhole mass.

The radial tidal acceleration is zero since  $\nu' = 0$ .

Acceleration felt by a traveller should less than the gravitational acceleration at earth surface,  $g_{earth}$ . The condition imposed by MT as [ for  $\nu' = 0$ ]

$$|\mathbf{f}| = |\sqrt{[1 - \frac{b(r)}{r}]} \beta' c^2| \leq g_{earth} \quad (21)$$

For the traveller's velocity  $v = constant$ , one finds that  $|\mathbf{f}| = 0$ . In our model the condition (21) is automatically satisfied, the traveller feels a zero gravitational acceleration.

Now we consider, the trip takes the time say less or equal to one year for both the traveller and the observer that stay at rest at the space stations  $l = -l_1$  and  $l = +l_2$  as [1]

$$\Delta\tau_{traveler} = \int_{-l_1}^{l_2} \frac{dl}{v\beta} \leq 1 \text{ year}$$

$$\Delta t_{spacestation} = \int_{-l_1}^{l_2} \frac{dl}{\sqrt{g_{tt}}v} \leq 1 \text{ year}.$$

For low velocity  $v \ll c$ , we have  $\beta \approx 1$  and with  $e^\nu = constant$ , the above expressions reduce to

$$\Delta\tau_{traveler} \approx \frac{2l}{v\beta} = \sqrt{g_{tt}} \Delta t_{spacestation}.$$

For  $a \gg r_0$ ,  $g_{tt} = (1 - \frac{D}{a^2}) \approx 1$ , so that  $\Delta\tau_{traveler} \approx \Delta t_{spacestation} \approx \frac{2a}{v}$ .

Here  $\frac{2a}{v} \approx 1 \text{ year} = 3.16 \times 10^7 s$ .

If the traversal velocity is  $v = .005c$ , the junction surface is at  $a = 7.9 \times 10^{10} m$ .

In conclusion, we have discussed some characteristics of the spherically symmetric wormhole solution in presence of C-field. We have established a matching of an interior solution with an exterior Schwarzschild solutions. How a massive body warps space would be visualized by the embedding a curved two dimensional surface in a three dimensional flat ( Euclidean ) space. In our model, the wormhole can be visualized as catenoid of revolution  $r = \sqrt{D} \cosh \frac{z}{\sqrt{D}}$ . According to MT [1], if wormhole travel is possible for human beings, the traveller's journey must satisfy three constraints: (i) the entire trip should require less than or of order 1 year as measured both by the traveller and by the peoples who live in the stations at  $l = l_2$  and  $l = -l_1$  (ii) the acceleration  $\mathbf{f}$ , felt by the traveller less than 1 earth gravity (iii) the tidal accelerations between various parts of the traveller's body less than 1 earth gravity. In our model all the three conditions have been verified. Thus the C-field generated wormholes are usable and traversable in practice.



One should be noted that there exists some regions in which C-field may play the role of exotic matter. The creation field C is depicted in fig.5.

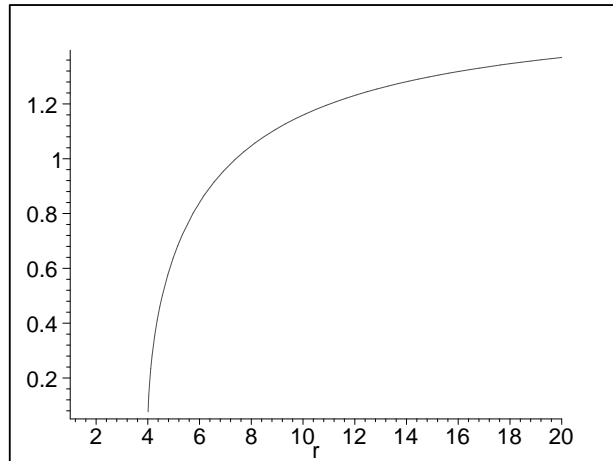


Figure 5: The diagram for the creation field C

Recently, a measure of quantifying exotic matter needed for traversable wormhole geometry has been developed within the framework of general relativity [19-20]. It is interesting to mention that no such quantifying measure is really needed in our model.

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